Topic Study Group 15

Teaching and Learning of Discrete Mathematics

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1. Aims of the TSG

Discrete mathematics is the study of discrete (as opposed to continuous) structures. It has many applications in a variety of fields, and it often exists at the interface of several disciplines, making it increasingly relevant in our digital world. Discrete mathematics offers many accessible points of entry for students to engage in rich mathematical thinking, as students can interact with ideas and reason about problems without requiring considerable prior knowledge of mathematical content. Further, its accessible nature makes it an excellent context in which students can engage in important mathematical practices such as conjecturing, generalizing, justifying, and proving. For these reasons, we view discrete mathematics as an indispensable part of mathematics education that deserves attention at all levels of education.

The main goal of TSG-15 was for researchers and educators to share current developments in the teaching and learning of discrete mathematics at all levels, ranging from elementary through postsecondary school. We sought to extend previous work on the teaching and learning of discrete mathematics by sharing new research and pedagogical innovations about a variety of topics related to discrete mathematics. We were particularly interested in identifying and exploring the variety of ways in which discrete mathematics is studied and taught across the world. We acknowledge that the teaching and learning of discrete mathematics may involve investigations into both mathematical content (particular mathematical topics within discrete mathematics and other disciplines) and mathematical practices (more general mathematical approaches or habits of mind), and that it may be a setting in which to explore other relevant issues in mathematics education. In terms of content, in this TSG we characterized discrete mathematics broadly as consisting of a variety of topics. This includes topics traditionally associated with discrete mathematics (such as algorithms, coding theory, combinatorics, cryptography, graph theory, languages and automata, logic, number theory, recursion, and set theory), as well as topics that might be considered relatively

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new (such as complexity theory, existence and constructability, and computational number theory, algebra, and group theory). In addition, there are a number of mathematical practices that could be related to discrete mathematics, including problem solving, conjecturing, justifying, generalizing, proving, and more. We also acknowledge that there may also be additional topics in mathematics education that might particularly be emphasized — these might include, for example, issues of affect and beliefs, equity and inclusion, classroom discourse, pre-service teacher preparation, or in-service teacher training.

We envisioned that this TSG would include presentations of papers on any of the wide range of topics discussed above, focusing on any level of school. We welcomed papers that were related to the teaching and learning of discrete mathematics, which may include, but were not limited to:

- research on student thinking about relevant concepts in discrete mathematics;
- research demonstrating effective instructional strategies in teaching discrete mathematics;
- research-based ideas for innovative activities and pedagogical interventions in classrooms at a variety of age levels;
- research-based ideas of incorporating technology into the discrete mathematics classroom;
- explorations of discrete mathematics as a setting in which to investigate mathematical practices;
- explorations of discrete mathematics as a setting in which to investigate other relevant issues in mathematics education;
- ways of thinking (or habits of mind) that may be productive in discrete mathematics, such as combinatorial reasoning, algorithmic or computational approaches, or recursive thinking;
- curriculum and educational policy issues related to discrete mathematics.

1.1. Submissions

We received 22 submissions from 12 countries (South America: 2; North America: 4; Asia: 3; Europe: 13). Of those 22 submissions, 18 were accepted as paper presentations, 2 as posters, and 2 were rejected. Four accepted paper presenters were not able to present in 2021 in the virtual format, so we had a total of 14 papers presented during the online conference (of these, one did not show up to the presentation, so there were 13 total presentations).

1.2. Sessions

In general, most presenters were given 20 total minutes, which included time for questions and the transition to the next speaker. Two exceptions were one 30-minute invited presentation (by Erik Tillema and Lori Burch[6]), and one shorter 10-minute presentation (who was a last-minute addition that had not indicated they intended to present). Our first session had four 20-minute presentations and one 10-minute presentation; our second session had one 30-minute presentation and three 20-minute presentations; and our third session had four 20-minute presentations. We also had some time at the end of the third session to have some overall discussion, although this did not seem like enough time to be able to have the kinds of in-depth conversations and discussions we would have liked to have. Still, there was a sense of community during the sessions, and even though we did not have much time for additional discussion, it felt like a productive and valuable shared experience.

1.3. Papers presented

Of the 18 accepted papers, four authors had to withdraw from the 2021 virtual participation. Thus, only 13 papers were able to be presented during the online conference. A list of these papers and authors are included in order of presentation and are organized in Tab. 1.

Tab. 1. List of papers presented

Paper and author(s) in order of presentation	
[1]	Suggestions for an integration of cryptology into a math curriculum. ***Tomas Borys*** (Germany)
[2]	Enriching pre-service teachers' conceptions about proof with discrete mathematics. ***Cécile Ouvrier-Buffet*** (France)
[3]	Graph theory in primary school mathematical education — a quantitative study on the impact of graph theory concepts on psychological characteristics of fourth grade students. ***Melissa Windler*** (Germany)
[4]	The role of discrete mathematics in secondary mathematics for non-STEM paths. ***Jaime Carvalho e Silva*** (Portugal)
[5]	Discrete mathematics in the Hungarian mathematics curriculum. ***Katalin Gosztonyi****, Csaba Csapodi, and Eötvös Loránd* (Hungary)
[6]	Leveraging combinatorial and quantitative reasoning to support the generalization of advanced algebraic identities. ***Erik Tillema*** *and Lori Burch* (U.S.A.)
[7]	Combinatorial counting problems in elementary school: a comparative analysis of German textbooks. ***Karina Höveler*** *and Janet Winzen* (Germany)
[8]	Preliminary levels of sophistication for enumerating permutations. ***Joseph Antonides*** *and Michael T. Battista* (U.S.A.)
[9]	Guiding students' reinvention of combinatorial operations. ***Belmiar Mota*** *and Rosa Antónia Tomás Ferreira* (Portugal)
[10]	Preservice teachers' development of mathematical knowledge for teaching via combinatorial tasks in a computational setting. ***Elise Lockwood*** *and Adaline De Chenne* (U.S.A.)
[11]	Relation between algorithmic and combinatorial thinking of undergraduate students of applied informatics. ***Janka Medová*** *and Sona Čeretková* (Slovakia)
[12]	Some approaches for incorporation of CAS in a discrete mathematics course. ***Mariana Durcheva*** (Bulgaria)
[13]	How can poly-universe sets develop creativity during the solution of combinatorial exercises? ***Eleonóra Stettner*** *and Szabina Tóth* (Hungary)

2. Conference Themes

We had presentations on a variety of topics, and we note some big ideas and themes that emerged during the sessions. These themes mostly align with areas of focus of our papers, and we describe categories of papers that highlight overall areas of emphasis that were covered in our topic study group.

One theme is that there are a number of ways in which discrete mathematics is and can be integrated in school mathematics. We saw examples of papers that demonstrated discrete mathematics for non-STEM majors (Carvalho e Silva[4]), for and for K-12 students in general (Gosztonyi et al.[5]). We also saw ideas for innovative ways to incorporate topics into the discrete mathematics curriculum (including Borys' focus on crytology[1]). This suggests that there are a variety of ways around the world in which discrete mathematics is being integrated into curricula, and this demonstrates that we have many opportunities to explore effective ways to teach discrete topics for students at a variety of age and grade levels.

A second theme is that the field is currently conducting (and would benefit from continuing to conduct) research about specific topics within discrete mathematics. The most common topic that is being regularly investigated is combinatorics, and eight of our papers focused on combinatorics (including the invited presentation by Tillema and Burch[6]). In addition, though, we saw promise for focusing on other topics, including graph theory (Windler[3]) and cryptology (Borys[1]). This suggests that there are opportunities for additional topics to be studied in more depth, and perhaps a next direction for the field is to jointly study other topics as thoroughly as combinatorics is being studied.

A third theme is that it may be productive to explore teacher preparation related to discrete mathematics. A couple of studies in our topic study group focused on pre-service teacher preparation (Ouvrier-Buffet[2]; Lockwood and De Chenne[10]), and this may continue to be a fertile area of research, where we may focus on the preparation of teachers who will teach topics in discrete mathematics. Particularly given the role of discrete mathematics in the curriculum in many different countries (as indicated in theme 1), it may be valuable to investigate in more depth how teachers are prepared to teach discrete topics.

Finally, our last theme is that discrete mathematics interfaces meaningfully with computing and technology. Several of our papers (including those by Medová and Čeretková[11]; Durcheva[12]) examined the role of technology and computing in discrete mathematics, suggesting that there may be valuable connections with researchers who study informatics or computer science. We see opportunities for interdisciplinarity in the future, particularly in this context of computing.

3. Areas for Future Research

There are several opportunities that we as a community identify as areas for future research, and our ideas for future research are related to the themes we described above. In particular, the distribution of topics about which we had presentations highlights different opportunities for more research in certain areas. While combinatorics has been increasingly well-researched in the last decades (which is underscored by our eight combinatorics-focused topics), other particular topics are as yet relatively under-researched. These include topics like cryptology and graph theory, as well as other areas of discrete mathematics like relations, sets, logic, and recursion. In terms of particular topics in discrete mathematics, then, there are many opportunities for researchers to investigate multiple aspects of these understudied topics, including students' reasoning about them and effective instructional interventions. We also hope that the field will continue to expand to investigate the teaching and learning of discrete mathematics as related to teacher preparation, exploring effective ways to prepare teachers to teach discrete topics in particular, and studying pre-service and in-service teachers' understandings of and experiences with discrete mathematics. Finally, there is a clear connection between discrete mathematics and computing, and we see valuable opportunities to continue to explore and examine the intersection of these ideas. We hope researchers will investigate effective ways to leverage computing in the teaching and learning of discrete mathematics, as well as ways to teach discrete mathematics to other populations such as informatics and computer science students.